# Test of the heavy quark-light diquark approximation for baryons with a heavy quark

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We check a commonly used approximation in which a baryon with a heavy quark is described as a heavy quark-light diquark system. The heavy quark influences the diquark internal motion reducing the average distance between the two light quarks. Besides, we show how the average distance between the heavy quark and any of the light quarks, and that between the heavy quark and the center of mass of the light diquark, are smaller than the distance between the two light quarks, which seems to contradict the heavy quark-light diquark picture. This latter result is in agreement with expectations from QCD sum rules and lattice QCD calculations. Our results also show that the diquark approximations produces larger masses than the ones obtained in a full calculation.

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## I. INTRODUCTION

Heavy quark symmetry [1, 2, 3, 4, 5, 6] (HQS) predicts that in baryons with a heavy quark, and up to corrections in the inverse of the heavy quark mass, the light degrees of freedom quantum numbers are well defined, in particular the total spin of the light degrees of freedom is well defined. This prediction has been taken in different calculations as the basis for treating the light quark subsystem as a diquark, and the baryon as a heavy quark-light diquark (HQLD) system [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. This HQS prediction does not imply though that the orbital motion of the two light quarks is not affected by the presence of the heavy quark as it seems to be implicit in the HQLD approximation [1]. Very recently the diquark structure of heavy baryons have been analyzed in $\Lambda_c$ production in heavy ion collisions [17] where its enhanced yield is seen as a signal for the existence of light diquark correlations both in the quark gluon plasma and the heavy baryon.

In Ref. [18], using a light-front constituent quark model and a Gaussian ansatz for the wave function, the authors studied the dependence of the Isgur-Wise function [4] on the baryon structure. They found very different behaviors for a diquark-like configuration (the heavy quark is far from the center of mass of the light quarks) or a collinear-type configuration (the heavy quark is close to the center of mass of the light quarks). Comparison of the results with QCD sum rules [19] and lattice QCD calculations [20] suggested a clear dominance of the collinear-type configurations. This result seems to go against the HQLD approximation.

Here we plan to check the validity of the HQLD approximation, that we formulate in next section, by looking at heavy baryons masses and quark distributions inside baryons composed of a heavy quark ($b$ or $c$) and two light quarks. We shall compare the predictions obtained within that approximation with the ones obtained in a full calculation where the effect of the heavy quark on the light diquark is not neglected. For that purpose we shall use the nonrelativistic quark model and the full wave functions described in Ref. [21]. In that reference we took advantage of HQS constraints on the spin of the light degrees of freedom to solve the full nonrelativistic three-body problem by means of a simple variational ansatz. The scheme of Ref. [21] for the wave functions reproduced previous results for masses, charge radii..., obtained in Ref. [22] by solving more involved Faddeev equations. The baryons included in that and the present study appear in Table I. We restrict ourselves to ground-state heavy baryons with total spin $J = 1/2, 3/2$ for which we could assume a zero total orbital angular momentum ($L = 0$).

## II. HEAVY QUARK-LIGHT DIQUARK APPROACH TO A HEAVY BARYON

The set of coordinates more adequate for a heavy quark-light diquark description are the Jacobi coordinates (See Fig. 1)

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[1] Note however that although in the HQLD approximation the light diquark internal structure is not affected by the heavy quark, this structure is commonly taken into account to build up the heavy quark-light diquark interaction.

| Baryon | $S$ | $J^P$ | $I$ | $S_l^\pi$ | Quark content | Baryon | $S$ | $J^P$ | $I$ | $S_l^\pi$ | Quark content |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Lambda_c$ | 0 | $\frac{1}{2}^+$ | 0 | $0^+$ | $udc$ | $\Lambda_b$ | 0 | $\frac{1}{2}^+$ | 0 | $0^+$ | $udb$ |
| $\Sigma_c$ | 0 | $\frac{1}{2}^+$ | 1 | $1^+$ | $llc$ | $\Sigma_b$ | 0 | $\frac{1}{2}^+$ | 1 | $1^+$ | $llb$ |
| $\Sigma_c^*$ | 0 | $\frac{3}{2}^+$ | 1 | $1^+$ | $llc$ | $\Sigma_b^*$ | 0 | $\frac{3}{2}^+$ | 1 | $1^+$ | $llb$ |
| $\Xi_c$ | $-1$ | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $0^+$ | $lsc$ | $\Xi_b$ | $-1$ | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $0^+$ | $lsb$ |
| $\Xi_c'$ | $-1$ | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $1^+$ | $lsc$ | $\Xi_b'$ | $-1$ | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $1^+$ | $lsb$ |
| $\Xi_c^*$ | $-1$ | $\frac{3}{2}^+$ | $\frac{1}{2}$ | $1^+$ | $lsc$ | $\Xi_b^*$ | $-1$ | $\frac{3}{2}^+$ | $\frac{1}{2}$ | $1^+$ | $lsb$ |
| $\Omega_c$ | $-2$ | $\frac{1}{2}^+$ | 0 | $1^+$ | $ssc$ | $\Omega_b$ | $-2$ | $\frac{1}{2}^+$ | 0 | $1^+$ | $ssb$ |
| $\Omega_c^*$ | $-2$ | $\frac{3}{2}^+$ | 0 | $1^+$ | $ssc$ | $\Omega_b^*$ | $-2$ | $\frac{3}{2}^+$ | 0 | $1^+$ | $ssb$ |

TABLE I: Summary of the quantum numbers of ground-state heavy baryons containing a single heavy quark. $I$, and $S_l^\pi$ are the isospin, and the spin parity of the light degrees of freedom and $S$, $J^P$ are the strangeness and the spin parity of the baryon. We also give the quark content where $l$ denotes a light quark of flavor $u$ or $d$.

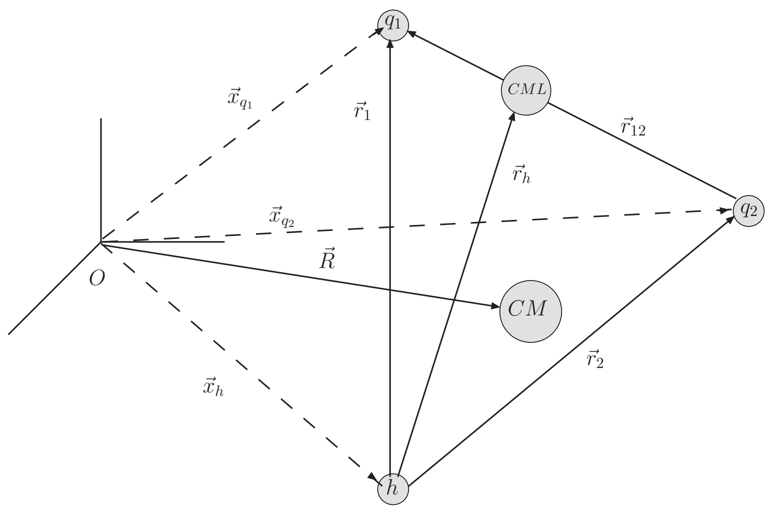


FIG. 1: Definition of different coordinates used through this work. $CM$ and $CML$ stand for the baryon center of mass and the light quark subsystem center of mass respectively.

$$\begin{aligned} \vec{R} &= \frac{m_{q_1}\vec{x}_{q_1} + m_{q_2}\vec{x}_{q_2} + m_h\vec{x}_h}{m_{q_1}+m_{q_2}+m_h} \\ \vec{r}_{12} &= \vec{x}_{q_1} - \vec{x}_{q_2} \\ \vec{r}_h &= \frac{m_{q_1}\vec{x}_{q_1} + m_{q_2}\vec{x}_{q_2}}{m_{q_1}+m_{q_2}} - \vec{x}_h \end{aligned} \tag{1}$$

where $\vec{x}_{q_1}$, $\vec{x}_{q_2}$ and $\vec{x}_h$ represent the positions, with respect to a certain reference frame, of the two light quarks and heavy quark respectively, and similarly $m_{q_1}$, $m_{q_2}$ and $m_h$ are their masses. The Jacobian coordinates are the center of mass position $\vec{R}$, the relative position between the two light quarks $\vec{r}_{12}$, and the relative position between the two light quark center of mass and the heavy quark $\vec{r}_h$.

In terms of these coordinates the three-body Hamiltonian can be written as

$$\begin{aligned} H &= -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}} + H^{\rm int} \quad ; \quad \overline{M} = m_{q_1}+m_{q_2}+m_h \\ H^{\rm int} &= \overline{M} + H_{q_1q_2} + H_{hq_1q_2} \end{aligned} \tag{2}$$

where $-\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}}$ accounts for the total center of mass free motion. Besides $\overline{M}$, the different terms in the internal Hamiltonian $H^{\rm int}$ are

$$H_{q_1q_2} = -\frac{\vec{\nabla}^2_{12}}{2\mu_{q_1q_2}} + V_{q_1q_2}(\vec{r}_{12},\, spin) \quad ; \quad \mu_{q_1q_2} = \frac{m_{q_1}m_{q_2}}{m_{q_1}+m_{q_2}}$$

$$H_{hq_1q_2} = -\frac{1}{2}\left(\frac{1}{m_{q_1}+m_{q_2}}+\frac{1}{m_h}\right)\vec{\nabla}_h^2 + V_{q_1h}(\vec{r}_h + \frac{m_{q_2}}{m_{q_1}+m_{q_2}}\vec{r}_{12},\, spin) + V_{q_2h}(\vec{r}_h - \frac{m_{q_1}}{m_{q_1}+m_{q_2}}\vec{r}_{12},\, spin) \tag{3}$$

with $\vec{\nabla}_{12}= \partial/\partial_{\vec{r}_{12}}$, $\vec{\nabla}_h= \partial/\partial_{\vec{r}_h}$ and $V_{qq'}$ the interquark potential that depends on relative distances and spins. Defining now

$$H^0_{hq_1q_2} = -\frac{1}{2}\left(\frac{1}{m_{q_1}+m_{q_2}}+\frac{1}{m_h}\right)\vec{\nabla}_h^2 + V_{q_1h}(\vec{r}_h,\, spin) + V_{q_2h}(\vec{r}_h,\, spin) \tag{4}$$

one could write

$$H^{\rm int} = \overline{M} + H_{q_1q_2} + H^0_{hq_1q_2} + (H_{hq_1q_2} - H^0_{hq_1q_2}) \tag{5}$$

$H_{q_1q_2}$ is the Hamiltonian for the relative motion of the two light quarks while $H^0_{hq_1q_2}$ is the Hamiltonian for the relative motion of the heavy quark with respect to a pointlike light diquark where the two light quarks are located in their center of mass. Both Hamiltonians are coupled through the term $(H_{hq_1q_2} - H^0_{hq_1q_2})$. This latter term can not be neglected altogether as the light diquark is not pointlike.

Within the HQLD approximation one assumes that the light diquark internal structure is not disturbed by the presence of the heavy quark. This means to neglect the influence of the term $(H_{hq_1q_2} - H^0_{hq_1q_2})$ in the evaluation of the diquark internal wave function, which therefore will be determined by $H_{q_1q_2}$ alone. However, and since the diquark will have a finite size, the effect of $(H_{hq_1q_2} - H^0_{hq_1q_2})$ has to be taken into account to obtain the $r_h$ dependence of the baryon wave function and its mass. Within this approximation, we will take a baryon wave function given by

$$\Psi^{B,\,HQLD}_{hq_1q_2}(r_{12}, r_h) = \Phi_{q_1q_2}(r_{12}) \cdot F_{hq_1q_2}(r_h) \tag{6}$$

where $\Phi_{q_1q_2}(r_{12})$ is the ground-state wave function for the Hamiltonian $H_{q_1q_2}$ and the given spin configuration. We will determine $F_{hq_1q_2}(r_h)$ variationally assuming an ansatz of the form

$$F_{hq_1q_2}(r_h) = \Phi^0_{hq_1q_2}(r_h) \cdot N\left(1 + \sum_{j=1}^{2} a_j\, e^{-b_j^2\,(r_h+c_j)^2}\right) \tag{7}$$

with $\Phi^0_{hq_1q_2}(r_h)$ the ground-state wave function for $H^0_{hq_1q_2}$ for the given spin configuration[2], and where $N$ is a normalization constant and $a_j,\, b_j,\, c_j;\ j = 1, 2$ are variational parameters that we determine by energy minimization. The variational parameters are compiled in the appendix.

## III. COMPARISON OF THE DIQUARK APPROXIMATION WITH THE FULL CALCULATION

As stated in the introduction, our full calculation in Ref. [21] took advantage of HQS constraints on the total spin of the light quarks to solve the full three-body problem by using a variational ansatz. All the information on the wave functions can be found there. In this section we shall compare results for masses and quark distributions obtained with the full calculation and with the HQLD approximation corresponding to Eqs. (6-7). All the results that we shall present have been obtained with the use of the AL1 interquark interaction of Refs. [22, 23]. This interaction contains a confinement term plus Coulomb and hyperfine terms coming from one gluon exchange. It was initially developed for mesons and for its use in baryons we have applied the usual $V_{qq} = V_{q\bar{q}}/2$ prescription [22, 24].

In Table II we compare the masses obtained as explained above. The masses of the full calculation are smaller in all cases, and thus better from a theoretical point of view[3]. They also compare better with experiment [25, 26, 27] and lattice estimates [28]. The results show that a full calculation makes the whole system more bound producing smaller masses.

The differences we have seen in the masses are a reflection of differences in the wave functions. We now make direct comparisons between the wave functions in the full calculation and in the HQLD approximation. We start

[2] $\Phi^0_{hq_1q_2}(r_h)$ and $\Phi_{q_1q_2}(r_{12})$ can be easily obtained by solving the corresponding Schrödinger equations with a Numerov algorithm.

[3] For a given Hamiltonian a variational wave function gives an upper limit to the ground-state mass

| Baryon | Full calcul. [21] | HQLD approx. | Exp. | Baryon | Full calcul. [21] | HQLD approx. | Exp. |
|---|---|---|---|---|---|---|---|
| $\Lambda_c$ | 2295 | 2317 | $2286.48 \pm 0.14$ | $\Lambda_b$ | 5643 | 5663 | $5624 \pm 9$ |
| $\Sigma_c$ | 2469 | 2521 | $2453.6 \pm 0.5$ | $\Sigma_b$ | 5851 | 5897 | $5812^{\S} \pm 3$ |
| $\Sigma_c^*$ | 2548 | 2579 | $2518 \pm 2$ | $\Sigma_b^*$ | 5882 | 5919 | $5833^{\S} \pm 3$ |
| $\Xi_c$ | 2474 | 2501 | $2469.5 \pm 0.6$ | $\Xi_b$ | 5808 | 5837 | $5760^{\ddagger} \pm 70$ |
| $\Xi_c'$ | 2578 | 2629 | $2577 \pm 4$ | $\Xi_b'$ | 5946 | 5993 | $5900^{\ddagger} \pm 70$ |
| $\Xi_c^*$ | 2655 | 2686 | $2646 \pm 1.4$ | $\Xi_b^*$ | 5975 | 6015 | $5900^{\ddagger} \pm 70$ |
| $\Omega_c$ | 2681 | 2727 | $2697.5 \pm 2.6$ | $\Omega_b$ | 6033 | 6081 | $5990^{\ddagger} \pm 70$ |
| $\Omega_c^*$ | 2755 | 2783 | $2768.3^{\dagger} \pm 3.2$ | $\Omega_b^*$ | 6063 | 6104 | $6000^{\ddagger} \pm 70$ |

TABLE II: Masses in MeV obtained with our full calculation in Ref. [21] and with the HQLD approximation (See text for details). In all cases we use the AL1 interquark potential of Refs. [22, 23]. We also show experimental masses (isospin average) and lattice estimates when the former are not known. Experimental masses have been taken from Refs. [25], [26] (†) and [27](§). Lattice estimates (‡) have been taken from Ref. [28]. Note in Ref. [26] what it is actually measured is the mass difference $M_{\Omega_c^*} - M_{\Omega_c}$.

by looking at the projection $\mathcal{P}$ of our full variational wave functions $\Psi^{B}_{hq_1q_2}(r_1, r_2, r_{12})$ obtained in Ref. [21] onto $\Psi^{B,\,HQLD}_{hq_1q_2}(r_{12}, r_h)$. Those projections are given by[4]

$$\mathcal{P} = \int d^3r_1 \int d^3r_2 \left(\Psi^{B}_{hq_1q_2}(r_1, r_2, r_{12})\right)^* \Psi^{B,\,HQLD}_{hq_1q_2}(r_{12}, r_h) \tag{8}$$

and the $|\mathcal{P}|^2$ values give an idea of how much of the "true" wave function is given by the wave function of the HQLD approximation. The values for $|\mathcal{P}|^2$ appear in Table III.

| | $\Lambda_c$ | $\Sigma_c$ | $\Sigma_c^*$ | $\Xi_c$ | $\Xi_c'$ | $\Xi_c^*$ | $\Omega_c$ | $\Omega_c^*$ |
|---|---|---|---|---|---|---|---|---|
| $\lvert\mathcal{P}\rvert^2$ | 0.971 | 0.943 | 0.957 | 0.949 | 0.926 | 0.932 | 0.935 | 0.961 |

| | $\Lambda_b$ | $\Sigma_b$ | $\Sigma_b^*$ | $\Xi_b$ | $\Xi_b'$ | $\Xi_b^*$ | $\Omega_b$ | $\Omega_b^*$ |
|---|---|---|---|---|---|---|---|---|
| $\lvert\mathcal{P}\rvert^2$ | 0.949 | 0.946 | 0.951 | 0.924 | 0.921 | 0.922 | 0.935 | 0.946 |

TABLE III: Absolute value square of the $\mathcal{P}$ projection coefficient defined in Eq. (8)

The values are generally higher for $c$-baryons than for the corresponding $b$-baryons. For a given isospin and strangeness the larger values occur for baryons with $S_l = 0$, and in the cases where $S_l = 1$ the projection is maximum for states with total angular momentum 3/2. To get more insight into what could be left in the 3-8% discrepancy that one observes we have evaluated different quark distributions.

It is very interesting to compare the probability $P_{q_1q_2}$ for the two light quarks to be found at a relative distance $r$. This probability is evaluated as

$$P_{q_1q_2}(r) = \int d^3r_1 \int d^3r_2 \;\delta(r_{12} - r)\left|\Psi^{B}_{hq_1q_2}(r_1, r_2, r_{12})\right|^2 \tag{9}$$

in the full calculation, and more simply as

$$P_{q_1q_2}(r)\big|_{HQLD} = 4\pi\, r^2\; |\Phi_{q_1q_2}(r)|^2 \tag{10}$$

in the HQLD approximation case[5].

The results of $P_{q_1q_2}(r)$ for different $b$- and $c$- ground-state heavy baryons appear in Fig. 2. In the HQLD approximation the results do not depend on the heavy quark mass, only on the quark content of the diquark and on the $S_l$ value. This feature is shared by the full calculation where one sees little dependence on the heavy quark mass.

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[4] Note in Ref. [21] the variational wave functions were obtained using a different set of coordinates which are $\vec{r}_{12}$, $\vec{r}_1 = \vec{r}_h + \frac{m_{q_2}}{m_{q_1}+m_{q_2}}\,\vec{r}_{12}$ and $\vec{r}_2 = \vec{r}_h - \frac{m_{q_1}}{m_{q_1}+m_{q_2}}\,\vec{r}_{12}$. $\vec{r}_1$ and $\vec{r}_2$ are the relative coordinates of the two light quarks with respect to the heavy quark. Besides note that $d^3r_1 d^3r_2 = d^3r_{12} d^3r_h$

[5] Note in the HQLD approximation $P_{q_1q_2}$ is totally independent of $F_{hq_1q_2}(r_h)$.

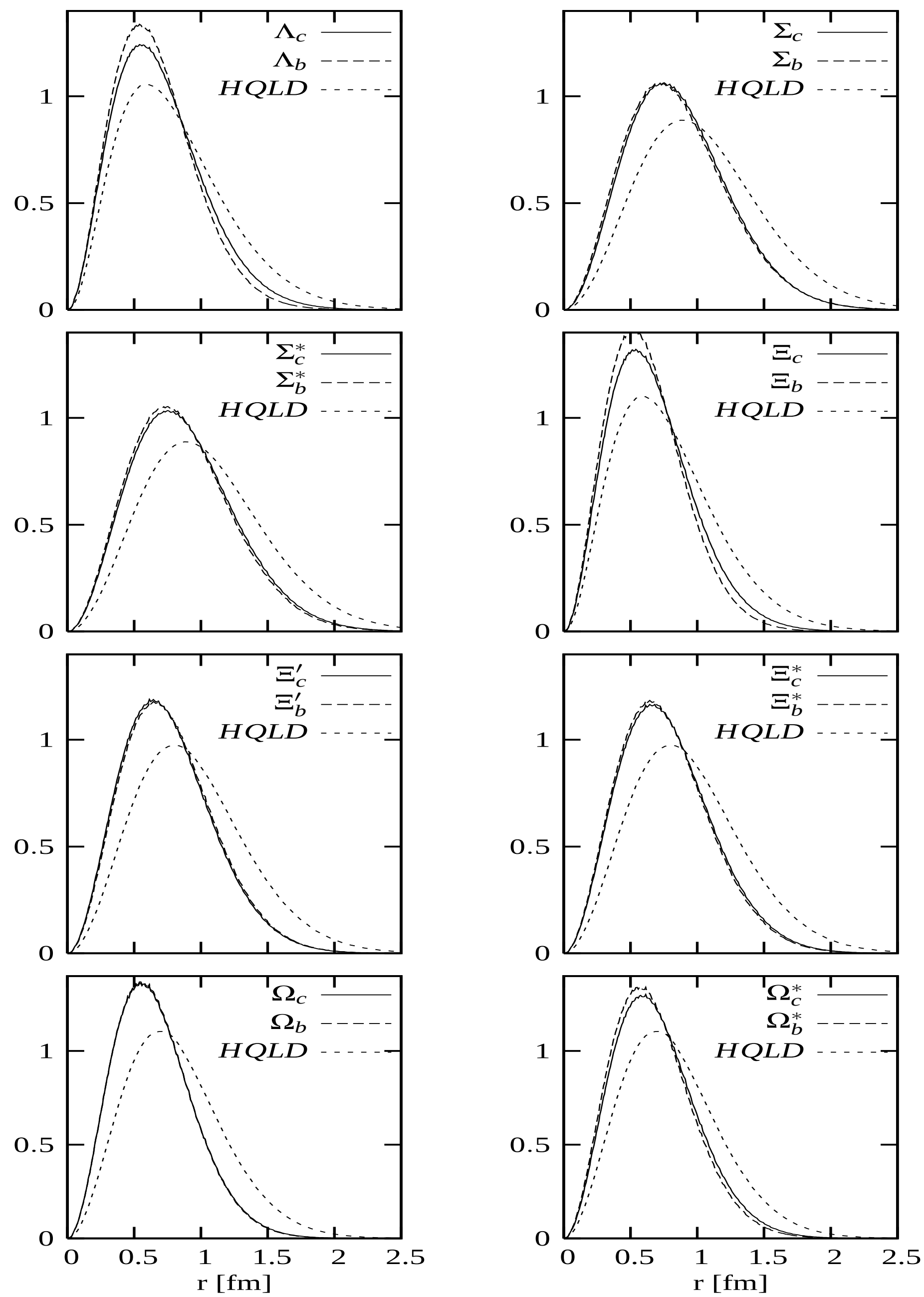


FIG. 2: $P_{q_1q_2}(r)$ defined in Eqs. (9-10) evaluated using our full calculation (solid lines for $c$-baryons and long-dashed lines for $b$-baryons) or with the HQLD approximation (short-dashed lines).

On the other hand we see in the full calculation how the presence of the heavy quark affects the diquark internal structure decreasing the relative distance between the two light quarks making the whole system more bound. This is the expected behavior from the comparison of the masses.

Another piece of information is provided by the full calculation probability $P_{hq_j}(r)$ to find the heavy quark at a certain distance $r$ of a light quark

$$P_{hq_j}(r) = \int d^3r_1 \int d^3r_2 \ \delta(r_j - r) \ \left|\Psi^B_{hq_1q_2}(r_1, r_2, r_{12})\right|^2 \tag{11}$$

The results are shown in Fig. 3 where for comparison we also show the corresponding $P_{q_1q_2}(r)$ distribution. From the figure one sees the heavy quark is closer to any of the two light quarks than the latter two among themselves.

Finally, we have also evaluated the probability distribution $P_{hCML}(r)$ for the heavy quark to be at a certain distance

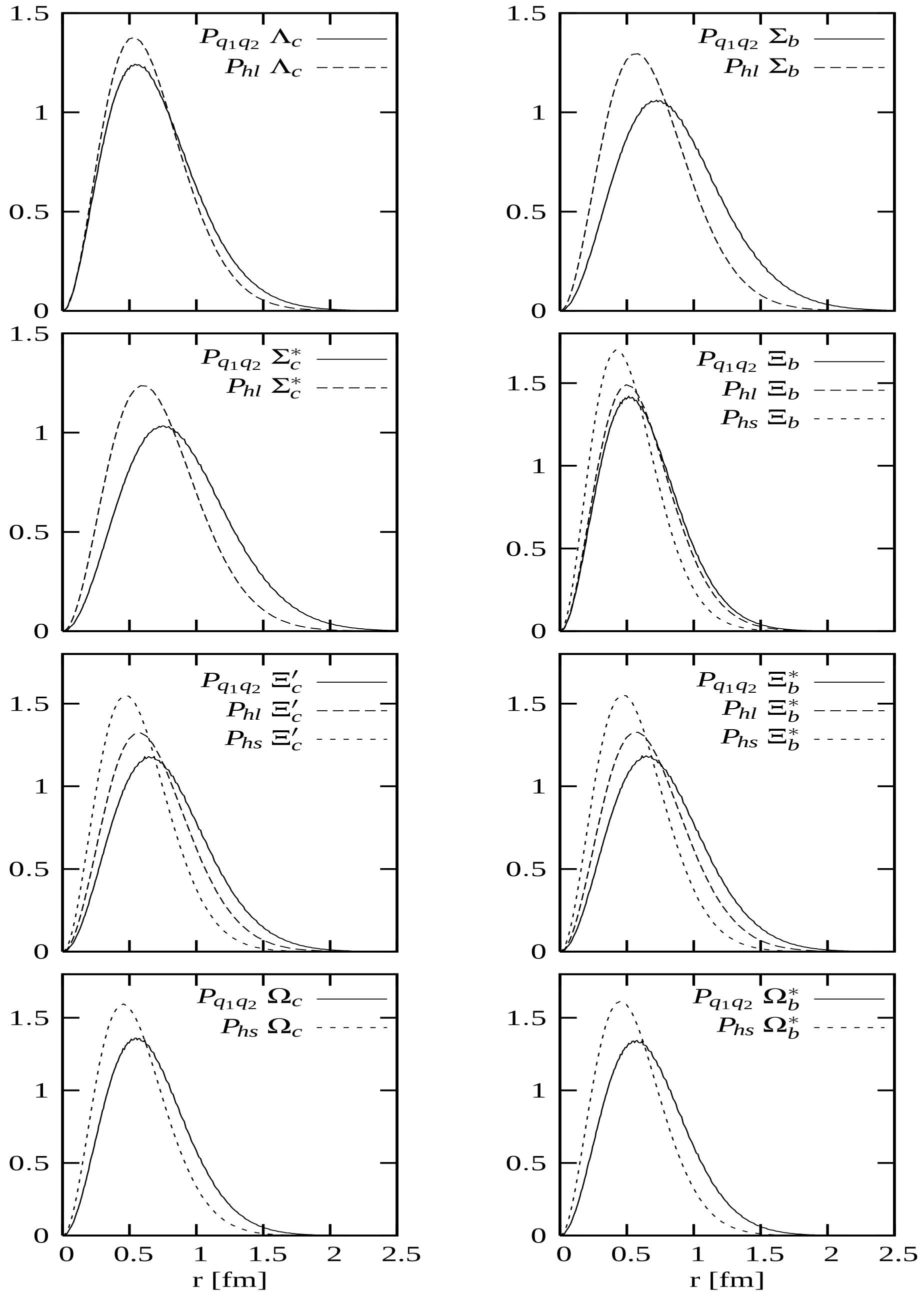


FIG. 3: $P_{hq_j}(r)$ (long-dashed lines for $q_j = l$ and short-dashed lines for $q_j = s$) and $P_{q_1q_2}(r)$ (solid lines) evaluated using our full calculation.

$r$ of the center of mass of the two light quarks $CML$. Again, this is simply given by

$$P_{hCML}(r) = \int d^3r_1 \int d^3r_2 \; \delta(r_h - r) \; \left|\Psi^B_{hq_1q_2}(r_1, r_2, r_{12})\right|^2 \tag{12}$$

and the results are shown in Fig. 4, where we also show the $P_{q_1q_2}(r)$ distributions. What one sees is that the average distance of the heavy quark to the center of mass of the light degrees of freedom is smaller than the average distance between the two light quarks.

The picture that emerges from this analysis is the one depicted in Fig.5, where the heavy quark is too close to the center of mass of the light degrees of freedom for the HQLD approximation to be fully valid. This result confirms the findings of Ref. [18]. There the comparison of the Isgur-Wise functions, obtained for different heavy baryon configurations, with the results of QCD sum rules [19] and lattice calculations [20] showed a dominance of this

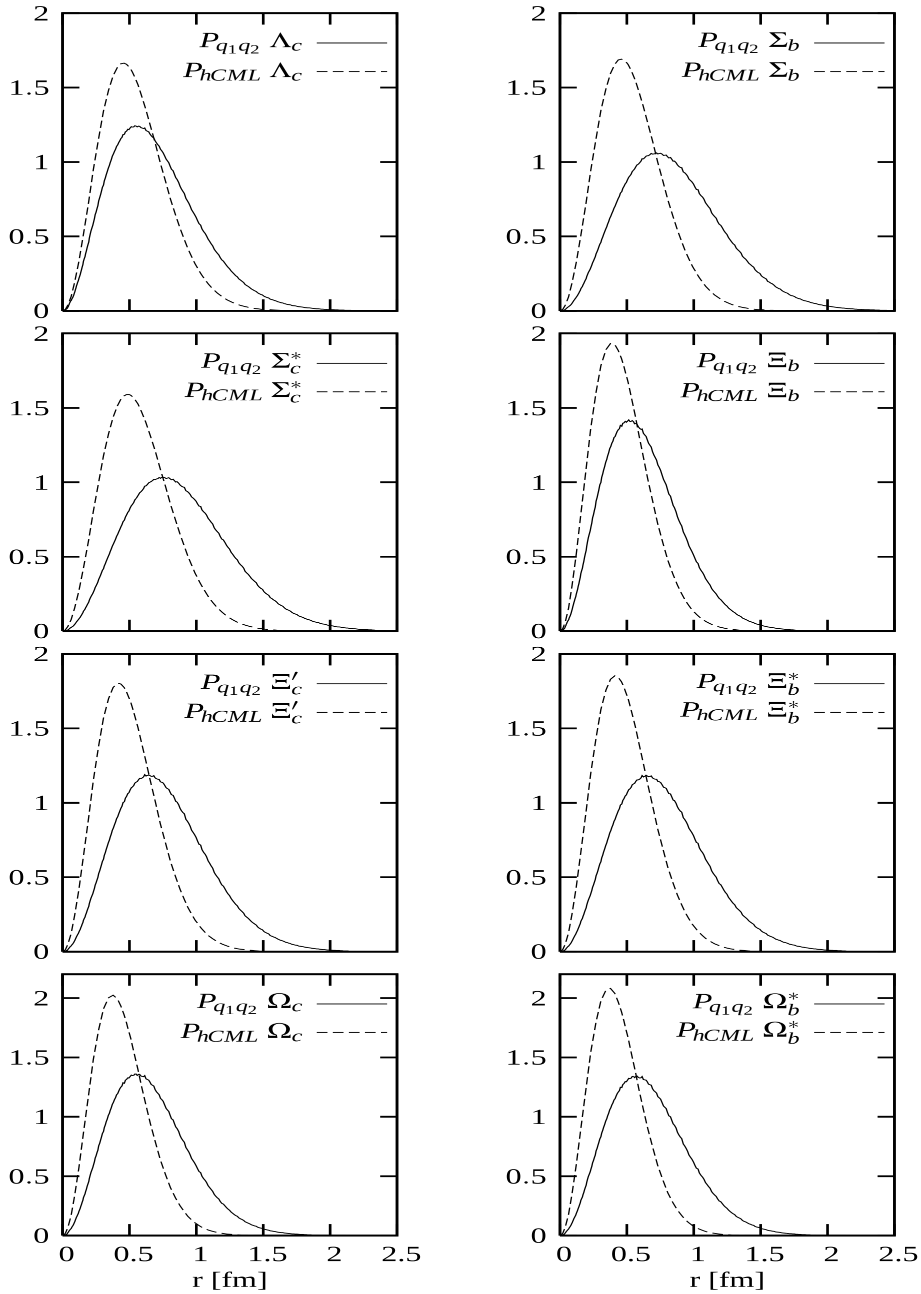


FIG. 4: $P_{hCML}(r)$ (dashed lines) and $P_{q_1q_2}(r)$ (solid lines) evaluated using our full calculation .

collinear-type configuration. By contrast, for doubly heavy baryons the light quark-heavy diquark picture is clearly favored. We illustrate this point in Fig. 6 for the case of doubly heavy $\Xi$ baryons. There we show the probability distribution $P_{h_1h_2}(r)$ for the two heavy quarks to be at a certain distance, and the probability distribution $P_{qCMH}(r)$ for the light quark to be found at a certain distance of the two heavy quark center of mass $CMH$. For the evaluation we have used our full wave functions obtained in Ref. [29]. We see how as the heavy quark masses increase the maximum of $P_{h_1h_2}(r)$ moves to lower distances while for $P_{qCMH}(r)$ the maximum does not change.

## IV. CONCLUDING REMARKS

We have checked the HQLD approximation by looking at its effects on masses and quark distributions inside the baryon. In that approximation the baryon is described as a bound state of a heavy quark and a light diquark which

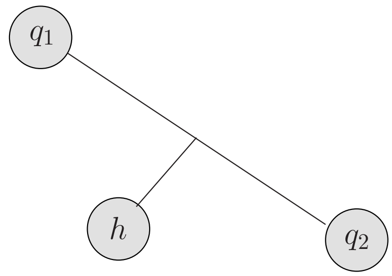


FIG. 5: Emerging schematic picture of a baryon with a heavy quark. This is in agreement wit the findings of Ref. [18] (See text for details).

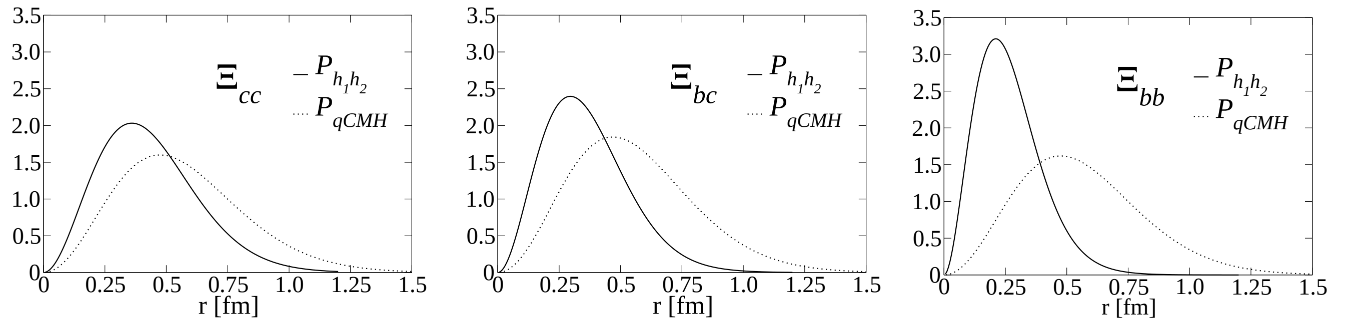


FIG. 6: Probability distribution $P_{h_1h_2}(r)$ (solid lines) for two heavy quarks to be found at a certain distance, and probability distribution $P_{qCMH}(r)$ (dotted lines) for the light quark to be found at a distance of the two heavy quark center of mass $CMH$ evaluated for doubly heavy $\Xi$ baryons. We have used our full wave functions from Ref. [29].

internal structure is not affected by the presence of the heavy quark. The approximation seems to work reasonably well at the level of total masses, although a full calculation produces smaller mass values. On the other hand our results show that the presence of the heavy quark affects notably the relative motion of the light degrees of freedom reducing the average distance between the two light quarks. Besides one sees that the heavy quark is closer to the light quarks than the latter among themselves, and that its average distance to the center of mass of the light quarks is also smaller than the size of the diquark. All this information seems to contradict the heavy quark-light diquark picture. Our study confirms previous analysis on the structure of heavy baryons done in Ref. [18]. Similar results concerning the quark distributions are obtained in the relativistic quark model of Ebert *et al.* [30]. The use of a full calculation seems to be preferable.


### Acknowledgments

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## APPENDIX A

In Table IV we give the values for the $a_j$, $b_j$, $c_j$; $j = 1, 2$ parameters of Eq. (7) that minimize the masses in the HQLD approximation.

| | $a_1$ | $b_1\,[fm^{-1}]$ | $c_1\,[fm]$ | $a_2$ | $b_2,[fm^{-1}]$ | $c_2\,[fm]$ | | $a_1$ | $b_1,[fm^{-1}]$ | $c_1\,[fm]$ | $a_2$ | $b_2,[fm^{-1}]$ | $c_2\,[fm]$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Lambda_c$ | −0.236 | 0.563 | 0.253 | −0.499 | 0.537 | 0.505 | $\Lambda_b$ | −0.493 | 0.524 | 0.921 | −0.427 | 0.524 | 0.998 |
| $\Sigma_c$ | −0.475 | 0.330 | 0.551 | −0.484 | 0.330 | 0.553 | $\Sigma_b$ | −0.376 | 0.211 | 0.851 | −0.618 | 0.190 | 1.014 |
| $\Sigma_c^*$ | −0.250 | 0.560 | 0.253 | −0.502 | 0.535 | 0.505 | $\Sigma_b^*$ | −0.384 | 0.397 | 0.822 | −0.560 | 0.380 | 0.980 |
| $\Xi_c$ | −0.589 | 0.544 | 0.917 | −0.330 | 0.555 | 0.998 | $\Xi_b$ | −0.456 | 0.507 | 0.797 | −0.511 | 0.497 | 0.965 |
| $\Xi_c'$ | −0.478 | 0.344 | 0.547 | −0.486 | 0.344 | 0.549 | $\Xi_b'$ | −0.477 | 0.392 | 0.521 | −0.489 | 0.391 | 0.533 |
| $\Xi_c^*$ | −0.569 | 0.499 | 0.928 | −0.348 | 0.507 | 1.019 | $\Xi_b^*$ | −0.359 | 0.421 | 0.769 | −0.634 | 0.398 | 0.958 |
| $\Omega_c$ | −0.474 | 0.396 | 0.516 | −0.489 | 0.395 | 0.532 | $\Omega_b$ | −0.487 | 0.408 | 0.528 | −0.491 | 0.408 | 0.529 |
| $\Omega_c^*$ | −0.435 | 0.583 | 0.856 | −0.525 | 0.573 | 0.967 | $\Omega_b^*$ | −0.360 | 0.419 | 0.697 | −0.652 | 0.393 | 0.925 |

TABLE IV:


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